

# Building LLM Reasoners Assignment 2: Systems and Parallelism

Spring 2026

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## 1 Assignment Overview

In this assignment, you will gain some hands-on experience with improving single-GPU training speed and scaling training to multiple GPUs.

**What you will implement.**

1. Benchmarking and profiling harness
2. Flash Attention 2 Triton kernel

**What the code looks like.** All the assignment code as well as this writeup are available on GitHub at:

<https://github.com/gregdurrett/nyu-llm-reasoners-a2>

Please `git clone` the repository. If there are any updates, we will notify you and you can `git pull` to get the latest.

1. `a1-basics/`: In this assignment, you'll be profiling some of the components that we built in assignment 1. This folder contains the staff solution code for assignment 1, so you will find a `a1-basics/pyproject.toml` and a `a1-basics/a1_basics/*` module in here. If you want to use your own implementation of the model, you can modify the `pyproject.toml` file in the base directory to point to your own package.
2. `/`: The base directory. We created an empty module named `student`. Note that there's no code in here, so you should be able to do whatever you want from scratch.
3. `tests/*.py`: This contains all the tests that you must pass. These tests invoke the hooks defined in `tests/adapters.py`. You'll implement the adapters to connect your code to the tests. Writing more tests and/or modifying the test code can be helpful for debugging your code, but your implementation is expected to pass the original provided test suite.
4. `README.md`: This file contains more details about the expected directory structure, as well as some basic instructions on setting up your environment.

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<sup>1</sup><https://stanford-cs336.github.io/spring2025/>

**How to submit.** You will submit the following files to Gradescope:

- `writeup.pdf`: Answer all the written questions. Please typeset your responses.
- `code.zip`: Contains all the code you've written. This should be a zipped file containing: (1) `student/` containing your implementation; (2) `tests/` containing just a single file `adapters.py` with your adapter code. **The autograder will check that the tests pass, but you should include all code that you wrote for the assignment, even code that is not tested by the autograder parts.**

**Doing the assignment** The majority of the exercises on this assignment require a GPU (almost everything except your PyTorch implementations of FlashAttention forward and backward). We recommend you run these with interactive jobs on GPU nodes; this will enable tight debugging loops.

Triton can be run with a CPU emulator, which is how the autograder runs it (`TRITON_INTERPRET=1`). However, Triton is not available for Mac; you will need to run these parts of the assignment on Linux.

In the first part of the assignment, we will look into how to optimize the performance of our Transformer model to make the most efficient use of the GPU. We will profile our model to understand where it spends time and memory during the forward and backward passes, then optimize the self-attention operation with custom GPU kernels, making it faster than a straightforward PyTorch implementation.

## 1.1 Profiling and Benchmarking

Before implementing any optimization, it is helpful to first profile our program to understand where it spends resources (e.g., time and memory). Otherwise, we risk optimizing parts of the model that don't account for significant time or memory, and therefore not seeing measurable end-to-end improvements.

We will implement three performance evaluation paths: (a) a simple, end-to-end benchmarking using the Python standard library to time our forward and backward passes, (b) profile compute with the NVIDIA Nsight Systems tool to understand how that time is distributed across operations on both the CPU and GPU, and (c) profile memory usage.

### 1.1.1 Setup - Importing your Basics Transformer Model

Let's start by making sure that you can load the model from the previous assignment. In the previous assignment, we set up our model in a Python package, so that it could be easily imported later. We have added the staff implementation of the model in the `./a1-basics` folder, and have pointed to it in the `pyproject.toml` file. By calling `uv run [command]` as usual, `uv` will automatically locate this local `a1-basics` package. If you would like to use your own implementation of the model, you can modify the `pyproject.toml` file to point to your own package.

You can test that you can import your model with:

```
1 ~$ uv run python
2 Using CPython 3.12.10
3 Creating virtual environment at: /path/to/uv/env/dir
4     Built student @ file:///path/to/student/dir
5     Built a1-basics @ file:///path/to/basics/dir
6 Installed 85 packages in 711ms
7 Python 3.12.10 (main, Apr  9 2025, 04:03:51) [Clang 20.1.0 ] on linux
8 ...
9 >>> import a1_basics
10 >>>
```

The relevant modules from assignment 1 should now be available (e.g., for `model.py`, you can import it with `import a1_basics.model`).

### 1.1.2 Model Sizing

Throughout this assignment, we will be benchmarking and profiling models to better understand their performance. To get a sense of how things change at scale, we will work with and refer to the following model configurations. For all models, we'll use a vocabulary size of 10,000 and a batch size of 4, with varying context lengths. This assignment (and later ones) will require a lot of results to be presented in tables. We strongly recommend that you automate constructing tables for your writeup in code, since formatting tables in LaTeX or Markdown can be very tedious. See `pandas.DataFrame.to_latex()` and `pandas.DataFrame.to_markdown()` or write your own function to generate them from your preferred tabular representation.

### 1.1.3 End-to-End Benchmarking

We will now implement a simple performance evaluation script. We will be testing many variations of our model (changing precision, swapping layers, etc.), so **it will pay off to have your script enable**

Size	d_model	d_ff	num_layers	num_heads
small	768	3072	12	12
medium	1024	4096	24	16
large	1280	5120	36	20
xl	1600	6400	48	25
2.7B	2560	10240	32	32

Table 1: Specifications of different model sizes

these variations via command-line arguments to make them easy to run later on. We also **highly recommend running sweeps over benchmarking hyperparameters, such as model size, context length, etc., using sbatch or submitit on Slurm** for quick iteration.

To start off, let’s do the simplest possible profiling of our model by timing the forward and backward passes. Since we will only be measuring speed and memory, we will use random weights and data.

Measuring performance is subtle — some common traps can cause us to not measure what we want. For benchmarking GPU code, one caveat is that CUDA calls are *asynchronous*. When you call a CUDA kernel, such as when you invoke `torch.matmul`, the function call returns control to your code without waiting for the matrix multiplication to finish. In this way, the CPU can continue running while the GPU computes the matrix multiplication. On the other hand, this means that naïvely measuring how long the `torch.matmul` call takes to return does not tell us how long the GPU takes to actually run the matrix multiplication. In PyTorch, we can call `torch.cuda.synchronize()` to wait for all GPU kernels to complete, allowing us to get more accurate measurements of CUDA kernel runtime. With this in mind, let’s write our basic profiling infrastructure.

**Problem (benchmarking\_script): 10 points**

- (a) Write a script to perform basic end-to-end benchmarking of the forward and backward passes in your model. Specifically, your script should support the following:
- Given hyperparameters (e.g., number of layers), initialize a model.
  - Generate a random batch of data.
  - Run  $w$  warm-up steps (before you start measuring time), then time the execution of  $n$  steps (either only forward, or both forward and backward passes, depending on an argument). For timing, you can use the Python `timeit` module (e.g., either using the `timeit` function, or using `timeit.default_timer()`, which gives you the system’s highest resolution clock, thus a better default for benchmarking than `time.time()`).
  - Call `torch.cuda.synchronize()` after each step.

**Deliverable:** A script that will initialize a `basics` Transformer model with the given hyperparameters, create a random batch of data, and time forward and backward passes.

- (b) Time the forward and backward passes for the model sizes described in §1.1.2. Use 5 warmup steps and compute the average and standard deviation of timings over 10 measurement steps. How long does a forward pass take? How about a backward pass? Do you see high variability across measurements, or is the standard deviation small?

**Deliverable:** A 1-2 sentence response with your timings.

- (c) One caveat of benchmarking is not performing the warm-up steps. Repeat your analysis without the warm-up steps. How does this affect your results? Why do you think this happens? Also try to run the script with 1 or 2 warm-up steps. Why might the result still be different?

**Deliverable:** A 2-3 sentence response.

#### 1.1.4 Nsight Systems Profiler

End-to-end benchmarking does not tell us where our model spends time and memory during forward and backward passes, and so does not expose specific optimization opportunities. To know how much time our program spends in each component (e.g., function), we can use a *profiler*. An execution profiler instruments the code by inserting guards when functions begin and finish running, and thus can give detailed execution statistics at the function level (such as number of calls, how long they take on average, cumulative time spent on this function, etc).

Standard Python profilers (e.g., `CProfile`) are not able to profile CUDA kernels since these kernels are executed asynchronously on the GPU. Fortunately, NVIDIA ships a profiler that we can use via the CLI `nsys`, which we have already installed for you. In this part of the assignment, you will use `nsys` to analyze the runtime of your Transformer model. Using `nsys` is straightforward: we can simply run your Python script from the previous section with `nsys profile` prepended. For example, you can profile a script `benchmark.py` and write the output to a file `result.nsys.rep` with:

```
1 ~$ uv run nsys profile -o result python benchmark.py
```

You can then view the profile on your local machine with the NVIDIA Nsight Systems desktop application. Selecting a particular CUDA API call (on the CPU) in the `CUDA API` row of the profile will highlight all corresponding kernel executions (on the GPU) in the `CUDA HW` row.

We encourage you to experiment with various command-line options for `nsys profile` to get a sense of what it can do. Notably, you can get Python backtraces for each CUDA API call with `--python-backtrace=cuda`, though this may introduce overhead. You can also annotate your code with NVTX ranges, which will appear as blocks in the `NVTX` row of the profile capturing all CUDA API calls and associated kernel executions. In particular, you should use NVTX ranges to **ignore the warm-up steps in your benchmarking script** (by applying a filter on the `NVTX` row in the profile). You can also isolate which kernels are responsible for the forward and backward passes of your model, and you can even isolate which kernels are responsible for different parts of a self-attention layer by annotating your implementation as follows:

```
1 ...
2 import torch.cuda.nvtx as nvtx
3
4 @nvtx.range("scaled dot product attention")
5 def annotated_scaled_dot_product_attention(
6     ... # Q, K, V, mask
7 )
8     ...
9     with nvtx.range("computing attention scores"):
10         ... # compute attention scores between Q and K
11
12     with nvtx.range("computing softmax")
13         ... # compute softmax of attention scores
14
15     with nvtx.range("final matmul")
16         ... # compute output projection
17
18     return ...
```

You can swap your original implementation with the annotated version in your benchmarking script via:

```
1 a1_basics.model.scaled_dot_product_attention = annotated_scaled_dot_product_attention
```

Finally, you can use the `--pytorch` command-line option with `nsys` to automatically annotate calls to the PyTorch C++ API with NVTX ranges.

### Problem (`nsys_profile`): 10 points

Profile your forward pass, backward pass, and optimizer step using `nsys` with each of the model sizes described in Table 1 and context lengths of 128, 256, 512 and 1024 (you may run out of memory with some of these context lengths for the larger models, in which case just note it in your report).

- (a) What is the total time spent on your forward pass? Does it match what we had measured before with the Python standard library?

**Deliverable:** A 1-2 sentence response.

- (b) What CUDA kernel takes the most cumulative GPU time during the forward pass? How many times is this kernel invoked during a single forward pass of your model? Is it the same kernel that takes the most runtime when you do both forward and backward passes? (Hint: look at the “CUDA GPU Kernel Summary” under “Stats Systems View”, and filter using NVTX ranges to identify which parts of the model are responsible for which kernels.)

**Deliverable:** A 1-2 sentence response.

- (c) Although the vast majority of FLOPs take place in matrix multiplications, you will notice that several other kernels still take a non-trivial amount of the overall runtime. What other kernels besides matrix multiplies do you see accounting for non-trivial CUDA runtime in the forward pass?

**Deliverable:** A 1-2 sentence response.

- (d) Profile running one complete training step with your implementation of AdamW (i.e., the forward pass, computing the loss and running a backward pass, and finally an optimizer step, as you’d do during training). How does the fraction of time spent on matrix multiplication change, compared to doing inference (forward pass only)? How about other kernels?

**Deliverable:** A 1-2 sentence response.

- (e) Compare the runtime of the softmax operation versus the matrix multiplication operations within the self-attention layer of your model during a forward pass. How does the difference in runtimes compare to the difference in FLOPs?

**Deliverable:** A 1-2 sentence response.

### 1.1.5 Mixed Precision

Up to this point in the assignment, we’ve been running with FP32 precision—all model parameters and activations have the `torch.float32` datatype. However, modern NVIDIA GPUs contain specialized GPU cores (Tensor Cores) for accelerating matrix multiplies at lower precisions. For example, the NVIDIA A100 spec sheet says that its maximum throughput with FP32 is 19.5 TFLOP/second, while its maximum throughput with FP16 (half-precision floats) or BF16 (brain floats) is significantly higher at 312 TFLOP/second. As a result, using lower-precision datatypes should help us speed up training and inference.

However, naïvely casting our model into a lower-precision format may come with reduced model accuracy. For example, many gradient values in practice are often too small to be representable in FP16, and thus become zero when naïvely training with FP16 precision. To combat this, it’s common to use loss scaling when training with FP16—the loss is simply multiplied by a scaling factor, increasing gradient magnitudes so they don’t flush to zero. Furthermore, FP16 has a lower dynamic range than FP32, which can lead to

overflows that manifest as a NaN loss. Full bfloat16 training is generally more stable (since BF16 has the same dynamic range as FP32), but can still affect final model performance compared to FP32.

To take advantage of the speedups from lower-precision datatypes, it's common to use *mixed-precision* training. In PyTorch, this is implemented with the `torch.autocast` context manager. In this case, certain operations (e.g., matrix multiplies) are performed in lower-precision datatypes, while other operations that require the full dynamic range of FP32 (e.g., accumulations and reductions) are kept as-is. For example, the following code will automatically identify which operations to perform in lower-precision during the forward pass and cast these operations to the specified data type:

```
1 model : torch.nn.Module = ... # e.g. your Transformer model
2 dtype : torch.dtype = ... # e.g. torch.float16
3 x : torch.Tensor = ... # input data
4
5 with torch.autocast(device="cuda", dtype=dtype):
6     y = model(x)
```

As alluded to above, it is generally a good idea to keep accumulations in higher precision even if the tensors themselves being accumulated have been downcasted. The following exercise will help build your intuition as to why this is the case.

#### Problem (mixed\_precision\_accumulation): 5 points

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Run the following code and comment on the (accuracy of the) results.

```
s = torch.tensor(0, dtype=torch.float32)
for i in range(1000):
    s += torch.tensor(0.01, dtype=torch.float32)
print(s)

s = torch.tensor(0, dtype=torch.float16)
for i in range(1000):
    s += torch.tensor(0.01, dtype=torch.float16)
print(s)

s = torch.tensor(0, dtype=torch.float32)
for i in range(1000):
    s += torch.tensor(0.01, dtype=torch.float16)
print(s)

s = torch.tensor(0, dtype=torch.float32)
for i in range(1000):
    x = torch.tensor(0.01, dtype=torch.float16)
    s += x.type(torch.float32)
print(s)
```

**Deliverable:** A 2-3 sentence response.

We will now apply mixed precision first to a toy model for intuition and then to our benchmarking script.

#### Problem (benchmarking\_mixed\_precision): 5 points

---

(a) Consider the following model:

```

1 class ToyModel(nn.Module):
2     def __init__(self, in_features: int, out_features: int):
3         super().__init__()
4         self.fc1 = nn.Linear(in_features, 10, bias=False)
5         self.ln = nn.LayerNorm(10)
6         self.fc2 = nn.Linear(10, out_features, bias=False)
7         self.relu = nn.ReLU()
8
9     def forward(self, x):
10        x = self.relu(self.fc1(x))
11        x = self.ln(x)
12        x = self.fc2(x)
13        return x

```

Suppose we are training the model on a GPU and that the model parameters are originally in FP32. We'd like to use autocasting mixed precision with FP16. What are the data types of:

- the model parameters within the autocast context,
- the output of the first feed-forward layer (`ToyModel.fc1`),
- the output of layer norm (`ToyModel.ln`),
- the model's predicted logits,
- the loss,
- and the model's gradients?

**Deliverable:** The data types for each of the components listed above.

- (b) You should have seen that FP16 mixed precision autocasting treats the layer normalization layer differently than the feed-forward layers. What parts of layer normalization are sensitive to mixed precision? If we use BF16 instead of FP16, do we still need to treat layer normalization differently? Why or why not?

**Deliverable:** A 2-3 sentence response.

- (c) Modify your benchmarking script to optionally run the model using mixed precision with BF16. Time the forward and backward passes with and without mixed-precision for each language model size described in §1.1.2. Compare the results of using full vs. mixed precision, and comment on any trends as model size changes. You may find the `nullcontext` no-op context manager to be useful.

**Deliverable:** A 2-3 sentence response with your timings and commentary.

### 1.1.6 Profiling Memory

So far, we have been looking at compute performance. We'll now shift our attention to *memory*, another major resource in language model training and inference. PyTorch also ships with a powerful memory profiler, which can keep track of allocations over time.

To use the memory profiler, you can modify your benchmarking script as follows:

```

1
2 ... # warm-up phase in your benchmarking script
3
4 # Start recording memory history.
5 torch.cuda.memory._record_memory_history(max_entries=1000000)

```

```

6
7 ... # what you want to profile in your benchmarking script
8
9 # Save a pickle file to be loaded by PyTorch's online tool.
10 torch.cuda.memory._dump_snapshot("memory_snapshot.pickle")
11
12 # Stop recording history.
13 torch.cuda.memory._record_memory_history(enabled=None)

```

This will output a file `memory_snapshot.pickle` that you can load into the following online tool: [https://pytorch.org/memory\\_viz](https://pytorch.org/memory_viz). This tool will let you see the overall memory usage timeline as well as each individual allocation that was made, with its size and a stack trace leading to the code where it originates. To use this tool, you should open the link above in a Web browser, and then drag and drop your Pickle file onto the page.

You will now use the PyTorch profiler to analyze the memory usage of your model.

### Problem (memory\_profiling): 8 points

Profile your forward pass, backward pass, and optimizer step of the 2.7B model from Table 1 with context lengths of 128, 256 and 512.

- (a) Add an option to your profiling script to run your model through the memory profiler. It may be helpful to reuse some of your previous infrastructure (e.g., to activate mixed-precision, load specific model sizes, etc). Then, run your script to get a memory profile of the 2.7B model when either doing inference only (just forward pass) or a full training step. How do your memory timelines look like? Can you tell which stage is running based on the peaks you see?

**Deliverable:** Two images of the “Active memory timeline” of a 2.7B model, from the `memory_viz` tool: one for the forward pass, and one for running a full training step (forward and backward passes, then optimizer step), and a 2-3 sentence response.

- (b) What is the peak memory usage of each context length when doing a forward pass? What about when doing a full training step?

**Deliverable:** A table with two numbers per context length.

- (c) Find the peak memory usage of the 2.7B model when using mixed-precision, for both a forward pass and a full optimizer step. Does mixed-precision significantly affect memory usage?

**Deliverable:** A 2-3 sentence response.

- (d) Consider the 2.7B model. At our reference hyperparameters, what is the size of a tensor of activations in the Transformer residual stream, in single-precision? Give this size in MB (i.e., divide the number of bytes by  $1024^2$ ).

**Deliverable:** A 1-2 sentence response with your derivation.

- (e) Now look closely at the “Active Memory Timeline” from `pytorch.org/memory_viz` of a memory snapshot of the 2.7B model doing a forward pass. When you reduce the “Detail” level, the tool hides the smallest allocations to the corresponding level (e.g., putting “Detail” at 10% only shows the 10% largest allocations). What is the size of the largest allocations shown? Looking through the stack trace, can you tell where those allocations come from?

**Deliverable:** A 1-2 sentence response.

## 1.2 Optimizing Attention with FlashAttention-2

### 1.2.1 Benchmarking PyTorch Attention

Your profiling likely suggests that there is an opportunity for optimization, both in terms of memory and compute, in your attention layers. At a high level, the attention operation consists of a matrix multiplication followed by softmax, then another matrix multiplication:

$$\text{Attention}(Q, K, V) = \text{softmax} \left( \text{mask} \left( \frac{Q^\top K}{\sqrt{d_k}} \right) \right) V \quad (1)$$

The naïve attention implementation needs to save attention score matrices of shape `seq_len × seq_len` for each batch/head element, which can grow very large with long sequence lengths, causing out-of-memory errors for any tasks with long inputs or outputs. We will implement an attention kernel following the FlashAttention-2 paper, which computes attention by tiles and avoids ever explicitly materializing the `seq_len × seq_len` attention score matrices, enabling scaling to much longer sequence lengths.

#### Problem (pytorch\_attention): 5 points

- (a) Benchmark your attention implementation at different scales. Write a script that will:
- (a) Fix the batch size to 8 and don't use multihead attention (i.e. remove the head dimension).
  - (b) Iterate through the cartesian product of [16, 32, 64, 128] for the head embedding dimension  $d_{\text{model}}$ , and [256, 1024, 4096, 8192, 16384] for the sequence length.
  - (c) Create random inputs  $Q, K, V$  for the appropriate size.
  - (d) Time 100 forward passes through attention using the inputs.
  - (e) Measure how much memory is in use before the backward pass starts, and time 100 backward passes.
  - (f) Make sure to warm up, and to call `torch.cuda.synchronize()` after each forward/backward pass.

Report the timings (or out-of-memory errors) you get for these configurations. At what size do you get out-of-memory errors? Do the accounting for the memory usage of attention in one of the smallest configurations you find that runs out of memory (you can use the equations for memory usage of Transformers from Assignment 1). How does the memory saved for backward change with the sequence length? What would you do to eliminate this memory cost?

**Deliverable:** A table with your timings, your working out for the memory usage, and a 1-2 paragraph response.

### 1.3 Benchmarking JIT-Compiled Attention

Since version 2.0, PyTorch also ships with a powerful just-in-time compiler that automatically tries to apply a number of optimizations to PyTorch functions: see [https://pytorch.org/tutorials/intermediate/torch\\_compile\\_tutorial.html](https://pytorch.org/tutorials/intermediate/torch_compile_tutorial.html) for an intro. In particular, it will try to automatically generate fused Triton kernels by dynamically analyzing your computation graph. The interface to use the PyTorch compiler is very simple. For instance, if we wanted to apply it to a single layer of our model, we can use:

```
1 layer = SomePyTorchModule(...)
2 compiled_layer = torch.compile(layer)
```

Now, `compiled_layer` functionally behaves just like `layer` (e.g., with its forward and backward passes). We can also compile our entire PyTorch model with `torch.compile(model)`, or even a Python function that calls PyTorch operations.

#### Problem (`torch_compile`): 5 points

- (a) Extend your attention benchmarking script to include a compiled version of your PyTorch implementation of attention, and compare its performance to the uncompiled version with the same configuration as the `pytorch_attention` problem above.

**Deliverable:** A table comparing your forward and backward pass timings for your compiled attention module with the uncompiled version from the `pytorch_attention` problem above.

- (b) Now, compile your entire Transformer model in your end-to-end benchmarking script. How does the performance of the forward pass change? What about the combined forward and backward passes and optimizer steps?

**Deliverable:** A table comparing your vanilla and compiled Transformer model.

Given the scaling behaviors we’ve seen with respect to the sequence length, we need significant improvements to handle large sequences. Even with `torch.compile`, the current implementation suffers from very poor memory access patterns at long sequence length. For that, we will write a Triton implementation of FlashAttention-2, where we’ll have significantly more control over how memory is accessed and when to compute what.

### 1.3.1 Example - Weighted Sum

To introduce what you’ll need to know about Triton and how it interoperates with PyTorch, we will work through an example kernel for a “weighted sum” operation. For further resources on getting up to speed with Triton, see Triton’s tutorials.

Given an input matrix  $X$ , we’ll multiply its entries by a column-wise weight vector  $w$ , and sum each row, giving us the matrix-vector product of  $X$  and  $w$ . We are going to work through the forward pass of this operation first, and then write the Triton kernel for the backward pass.

**Forward pass** The forward pass of our kernel is just the following broadcasted inner product.

```
1 def weighted_sum(x, weight):
2     # Here, assume that x has n-dim shape [..., D], and weight has 1D shape [D]
3     return (weight * x).sum(axis=-1)
```

When writing our Triton kernel, we’ll have each program instance (potentially running in parallel) compute the weighted sum of a *tile* of rows of  $x$ , and write the corresponding scalar outputs to the output tensor. In Triton, a program instance is a block of threads all running the same program, and these *thread blocks* can be run in parallel on the GPU. Instead of taking *tensors* as arguments, we take *pointers* to their first elements, as well as *strides* for each tensor that tell us how to move along axes.

We can use the strides to load a tensor corresponding to the tile of rows of  $x$  that we’re summing in the running instance, using the program ID to divide up the work (i.e., instance  $i$  will process the  $i$ -th tile of rows of  $x$ ). The main difference between the forward pass in Triton and PyTorch in this simple case is the need to do pointer arithmetic and explicit loads/stores. We will use the block pointer abstraction with `tl.make_block_ptr` to greatly simplify the pointer arithmetic, although this means we need to do some setup to prepare the block pointers.

Refer to Figure 1 for a schematic of tiling and how block pointers are advanced. The weighted sum function from above looks like the following:

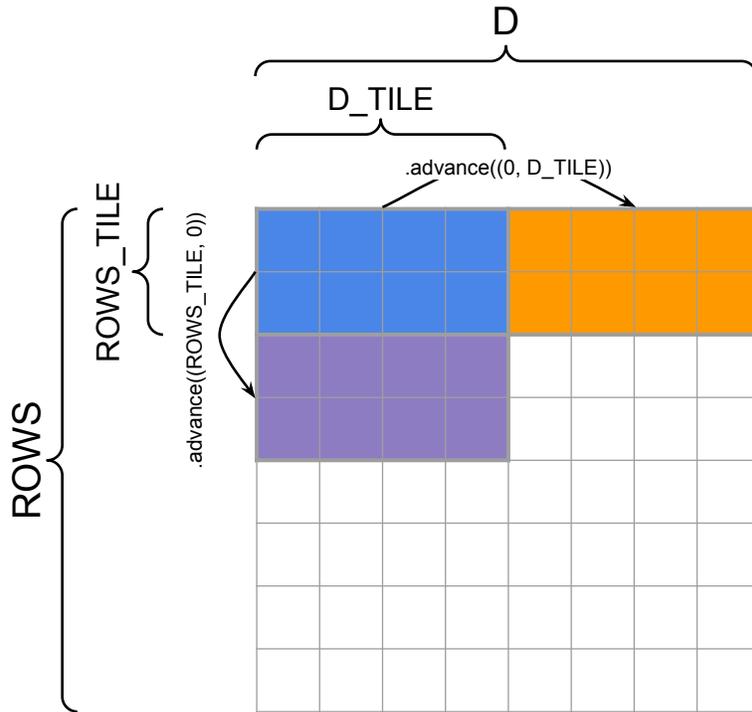


Figure 1: Tiling and advancing block pointers in the weighted sum kernel example (Section 1.3.1).

```

1  import triton
2  import triton.language as tl
3
4  @triton.jit
5  def weighted_sum_fwd(
6      x_ptr, weight_ptr, # Input pointers
7      output_ptr, # Output pointer
8      x_stride_row, x_stride_dim, # Strides tell us how to move one element in each axis of a tensor
9      weight_stride_dim, # Likely 1
10     output_stride_row, # Likely 1
11     ROWS, D,
12     ROWS_TILE_SIZE: tl.constexpr, D_TILE_SIZE: tl.constexpr, # Tile shapes must be known at compile time
13 ):
14     # Each instance will compute the weighted sum of a tile of rows of x.
15     # `tl.program_id` gives us a way to check which thread block we're running in
16     row_tile_idx = tl.program_id(0)
17
18     # Block pointers give us a way to select from an ND region of memory
19     # and move our selection around.
20     # The block pointer must know:
21     # - The pointer to the first element of the tensor
22     # - The overall shape of the tensor to handle out-of-bounds access
23     # - The strides of each dimension to use the memory layout properly
24     # - The ND coordinates of the starting block, i.e., "offsets"
25     # - The block shape to use load/store at a time
26     # - The order of the dimensions in memory from major to minor
27     # axes (= np.argsort(strides)) for optimizations, especially useful on H100
28

```

```

29     x_block_ptr = tl.make_block_ptr(
30         x_ptr,
31         shape=(ROWS, D),
32         strides=(x_stride_row, x_stride_dim),
33         offsets=(row_tile_idx * ROWS_TILE_SIZE, 0),
34         block_shape=(ROWS_TILE_SIZE, D_TILE_SIZE),
35         order=(1, 0),
36     )
37
38     weight_block_ptr = tl.make_block_ptr(
39         weight_ptr,
40         shape=(D,),
41         strides=(weight_stride_dim,),
42         offsets=(0,),
43         block_shape=(D_TILE_SIZE,),
44         order=(0,),
45     )
46
47     output_block_ptr = tl.make_block_ptr(
48         output_ptr,
49         shape=(ROWS,),
50         strides=(output_stride_row,),
51         offsets=(row_tile_idx * ROWS_TILE_SIZE,),
52         block_shape=(ROWS_TILE_SIZE,),
53         order=(0,),
54     )
55
56     # Initialize a buffer to write to
57     output = tl.zeros((ROWS_TILE_SIZE,), dtype=tl.float32)
58
59     for i in range(tl.cdiv(D, D_TILE_SIZE)):
60         # Load the current block pointer
61         # Since ROWS_TILE_SIZE might not divide ROWS, and D_TILE_SIZE might not divide D,
62         # we need boundary checks for both dimensions
63         row = tl.load(x_block_ptr, boundary_check=(0, 1), padding_option="zero") # (ROWS_TILE_SIZE, D_TILE_SIZE)
64         weight = tl.load(weight_block_ptr, boundary_check=(0,), padding_option="zero") # (D_TILE_SIZE,)
65
66         # Compute the weighted sum of the row.
67         output += tl.sum(row * weight[None, :], axis=1)
68
69         # Move the pointers to the next tile.
70         # These are (rows, columns) coordinate deltas
71         x_block_ptr = x_block_ptr.advance((0, D_TILE_SIZE)) # Move by D_TILE_SIZE in the last dimension
72         weight_block_ptr = weight_block_ptr.advance((D_TILE_SIZE,)) # Move by D_TILE_SIZE
73
74         # Write output to the output block pointer (a single scalar per row).
75         # Since ROWS_TILE_SIZE might not divide ROWS, we need boundary checks
76         tl.store(output_block_ptr, output, boundary_check=(0,))

```

Let's now wrap this kernel in a PyTorch Autograd function, that will interoperate with PyTorch (i.e., take Tensors as inputs, output a Tensor, and later also work with the autograd engine during the backward pass):

```

1 class WeightedSumFunc(torch.autograd.Function):
2     @staticmethod
3     def forward(ctx, x, weight):
4         # Cache x and weight to be used in the backward pass, when we
5         # only receive the gradient wrt. the output tensor, and
6         # need to compute the gradients wrt. x and weight.
7         D, output_dims = x.shape[-1], x.shape[:-1]
8
9         # Reshape input tensor to 2D
10        input_shape = x.shape
11        x = rearrange(x, "... d -> (...) d")
12
13        ctx.save_for_backward(x, weight)

```

```

14
15     assert len(weight.shape) == 1 and weight.shape[0] == D, "Dimension mismatch"
16     assert x.is_cuda and weight.is_cuda, "Expected CUDA tensors"
17     assert x.is_contiguous(), "Our pointer arithmetic will assume contiguous x"
18
19     ctx.D_TILE_SIZE = triton.next_power_of_2(D) // 16 # Roughly 16 loops through the embedding dimension
20     ctx.ROWS_TILE_SIZE = 16 # Each thread processes 16 batch elements at a time
21     ctx.input_shape = input_shape
22
23     # Need to initialize empty result tensor. Note that these elements are not necessarily 0!
24     y = torch.empty(output_dims, device=x.device)
25
26     # Launch our kernel with n instances in our 1D grid.
27     n_rows = y.numel()
28     weighted_sum_fwd[(cddiv(n_rows, ctx.ROWS_TILE_SIZE),)](
29         x, weight,
30         y,
31         x.stride(0), x.stride(1),
32         weight.stride(0),
33         y.stride(0),
34         ROWS=n_rows, D=D,
35         ROWS_TILE_SIZE=ctx.ROWS_TILE_SIZE, D_TILE_SIZE=ctx.D_TILE_SIZE,
36     )
37
38     return y.view(input_shape[:-1])

```

Notice that when we invoke the Triton kernel with `weighted_sum_fwd[(cddiv(n_rows, ctx.ROWS_TILE_SIZE),)]`, we define a so-called “launch grid” of thread blocks by passing the tuple `(cddiv(n_rows, ctx.ROWS_TILE_SIZE),)`. Then, we can access the thread block index with `tl.program_id(0)` in our kernel.

**Backward pass** Since we are defining our own kernel, we will also need to write our own backward function.

In the forward pass, we were given the inputs to our layer, and needed to compute its outputs. In the backward pass, recall that we will be given the gradients of the objective with respect to our outputs, and need to compute the gradient with respect to each of our inputs. In our case, our operation has as inputs a matrix  $x : \mathbb{R}^{n \times h}$  and a weight vector  $w : \mathbb{R}^h$ . For short, let’s call our operation  $f(x, w)$ , whose range is  $\mathbb{R}^n$ . Then, assuming we are given  $\nabla_{f(x,w)} \mathcal{L}$ , the gradient of loss  $\mathcal{L}$  with respect to the output of our layer, we can apply the multivariate chain rule to obtain the following expressions for the gradients with respect to  $x$  and  $w$ :

$$(\nabla_x \mathcal{L})_{ij} = \sum_{k=1}^n \frac{\partial f(x, w)_k}{\partial x_{ij}} (\nabla_{f(x,w)} \mathcal{L})_k = w_j \cdot (\nabla_{f(x,w)} \mathcal{L})_i \quad (2)$$

$$(\nabla_w \mathcal{L})_j = \sum_{i=1}^n \frac{\partial f(x, w)_i}{\partial w_j} (\nabla_{f(x,w)} \mathcal{L})_i = \sum_{i=1}^n x_{ij} \cdot (\nabla_{f(x,w)} \mathcal{L})_i \quad (3)$$

This gives a simple formula for computing the backward pass. To obtain the backward step with respect to  $x$ , we apply Eq 2 and take the outer product of  $w$  and  $\nabla_{f(x,w)}$ . To compute the backward step with respect to  $w$  (i.e.  $(\nabla_w \mathcal{L})_j$ ), we must multiply our input gradient by the corresponding output row.

Our kernel for the backward pass will start by defining all the block pointers and then computing  $\nabla_x \mathcal{L}$ :

```

1 @triton.jit
2 def weighted_sum_backward(
3     x_ptr, weight_ptr, # Input
4     grad_output_ptr, # Grad input
5     grad_x_ptr, partial_grad_weight_ptr, # Grad outputs
6     stride_xr, stride_xd,
7     stride_wd,
8     stride_gr,

```

```

9     stride_gxr, stride_gxd,
10    stride_gwb, stride_gwd,
11    NUM_ROWS, D,
12    ROWS_TILE_SIZE: tl.constexpr, D_TILE_SIZE: tl.constexpr,
13 ):
14     row_tile_idx = tl.program_id(0)
15     n_row_tiles = tl.num_programs(0)
16
17     # Inputs
18     grad_output_block_ptr = tl.make_block_ptr(
19         grad_output_ptr,
20         shape=(NUM_ROWS,), strides=(stride_gr,),
21         offsets=(row_tile_idx * ROWS_TILE_SIZE,),
22         block_shape=(ROWS_TILE_SIZE,),
23         order=(0,),
24     )
25
26     x_block_ptr = tl.make_block_ptr(
27         x_ptr,
28         shape=(NUM_ROWS, D), strides=(stride_xr, stride_xd),
29         offsets=(row_tile_idx * ROWS_TILE_SIZE, 0),
30         block_shape=(ROWS_TILE_SIZE, D_TILE_SIZE),
31         order=(1, 0),
32     )
33
34     weight_block_ptr = tl.make_block_ptr(
35         weight_ptr,
36         shape=(D,), strides=(stride_wd,),
37         offsets=(0,), block_shape=(D_TILE_SIZE,),
38         order=(0,),
39     )
40
41     grad_x_block_ptr = tl.make_block_ptr(
42         grad_x_ptr,
43         shape=(NUM_ROWS, D), strides=(stride_gxr, stride_gxd),
44         offsets=(row_tile_idx * ROWS_TILE_SIZE, 0),
45         block_shape=(ROWS_TILE_SIZE, D_TILE_SIZE),
46         order=(1, 0),
47     )
48
49     partial_grad_weight_block_ptr = tl.make_block_ptr(
50         partial_grad_weight_ptr,
51         shape=(n_row_tiles, D), strides=(stride_gwb, stride_gwd),
52         offsets=(row_tile_idx, 0),
53         block_shape=(1, D_TILE_SIZE),
54         order=(1, 0),
55     )
56
57     for i in range(tl.cdiv(D, D_TILE_SIZE)):
58         grad_output = tl.load(grad_output_block_ptr, boundary_check=(0,), padding_option="zero") # (ROWS_TILE_SIZE,)
59
60         # Outer product for grad_x
61         weight = tl.load(weight_block_ptr, boundary_check=(0,), padding_option="zero") # (D_TILE_SIZE,)
62         grad_x_row = grad_output[:, None] * weight[None, :]
63         tl.store(grad_x_block_ptr, grad_x_row, boundary_check=(0, 1))
64
65         # Reduce as many rows as possible for the grad_weight result
66         row = tl.load(x_block_ptr, boundary_check=(0, 1), padding_option="zero") # (ROWS_TILE_SIZE, D_TILE_SIZE)
67         grad_weight_row = tl.sum(row * grad_output[:, None], axis=0, keep_dims=True)
68         tl.store(partial_grad_weight_block_ptr, grad_weight_row, boundary_check=(1,)) # Never out of bounds for dim 0
69
70         # Move the pointers to the next tile along D
71         x_block_ptr = x_block_ptr.advance((0, D_TILE_SIZE))
72         weight_block_ptr = weight_block_ptr.advance((D_TILE_SIZE,))
73         partial_grad_weight_block_ptr = partial_grad_weight_block_ptr.advance((0, D_TILE_SIZE))
74         grad_x_block_ptr = grad_x_block_ptr.advance((0, D_TILE_SIZE))

```

Computing the gradient  $\nabla_x$  is simple, and we write the result to the appropriate tile of the output tensor. However, computing  $\nabla_w$  is a bit more challenging. Each kernel instance is responsible for one row tile of  $x$ , but we now need to sum *across* rows of  $x$ . Instead of doing this sum directly in our backward pass, we will assume that `partial_grad_weight_ptr` contains an `n_row_tiles × H` matrix, where the first dimension is only reduced within a row tile from  $x$ . We reduce within the current row tile before writing to this tensor. Outside of the kernel, we reduce  $\nabla_w$  using `torch.sum` to sum up the results from each row tile<sup>2</sup>. The final part of the `autograd.Function` is then relatively simple:

```

1 class WeightedSumFunc(torch.autograd.Function):
2     @staticmethod
3     def forward(ctx, x, weight):
4         # ... (defined earlier)
5
6     @staticmethod
7     def backward(ctx, grad_out):
8         x, weight = ctx.saved_tensors
9         ROWS_TILE_SIZE, D_TILE_SIZE = ctx.ROWS_TILE_SIZE, ctx.D_TILE_SIZE # These don't have to be the same
10        n_rows, D = x.shape
11
12        # Our strategy is for each thread block to first write to a partial buffer,
13        # then we reduce over this buffer to get the final gradient.
14        partial_grad_weight = torch.empty((cdiv(n_rows, ROWS_TILE_SIZE), D), device=x.device, dtype=x.dtype)
15        grad_x = torch.empty_like(x)
16
17        weighted_sum_backward[(cdiv(n_rows, ROWS_TILE_SIZE),)](
18            x, weight,
19            grad_out,
20            grad_x, partial_grad_weight,
21            x.stride(0), x.stride(1),
22            weight.stride(0),
23            grad_out.stride(0),
24            grad_x.stride(0), grad_x.stride(1),
25            partial_grad_weight.stride(0), partial_grad_weight.stride(1),
26            NUM_ROWS=n_rows, D=D,
27            ROWS_TILE_SIZE=ROWS_TILE_SIZE, D_TILE_SIZE=D_TILE_SIZE,
28        )
29        grad_weight = partial_grad_weight.sum(axis=0)
30        return grad_x, grad_weight

```

Finally, we can now obtain a function that works much like those implemented in `torch.nn.functional`:

```

1 f_weightedsum = WeightedSumFunc.apply

```

Now, calling `f_weightedsum` on two PyTorch tensors  $x$  and  $w$  will give a tensor such as the following:

```

1 tensor([ 90.8563, -93.6815, -80.8884, ..., 103.4840, -21.4634, -24.0192],
2        device='cuda:0', grad_fn=<WeightedSumFuncBackward>)

```

Note the `grad_fn` attached to the tensor — this shows that PyTorch knows what to call in the backward pass when this tensor appears in the computation graph. This completes our Triton implementation of the weighted sum operation.

### 1.3.2 FlashAttention-2 Forward Pass

You will replace your PyTorch attention implementation with a significantly improved Triton implementation following FlashAttention-2 [Dao, 2023]. FlashAttention-2 employs some tricks to compute the forward pass in tiles, which allows for efficient memory access patterns and avoids the need to materialize the full attention matrix on global memory.

Before jumping into this section, we highly recommend reading at least the original FlashAttention paper [Dao et al., 2022], which will give you intuition for the core technique that enables efficient attention with

<sup>2</sup>Or, of course, we could write our own kernel for that.

FlashAttention: computing the softmax in an online fashion across tiles (a technique proposed in [Milakov and Gimelshein, 2018]). We also recommend checking out He [2022] for some more intuition on how GPUs actually execute PyTorch code.

**Understanding inefficiencies in vanilla attention.** Recall that the forward pass for attention (ignoring masking for now) can be written as:

$$\mathbf{S} = \mathbf{Q}\mathbf{K}^\top / \sqrt{d} \tag{4}$$

$$\mathbf{P}_{ij} = \text{softmax}_j(\mathbf{S})_{ij} \tag{5}$$

$$\mathbf{O} = \mathbf{P}\mathbf{V} \tag{6}$$

The standard backward pass is

$$d\mathbf{V} = \mathbf{P}^\top d\mathbf{O} \tag{7}$$

$$d\mathbf{P} = d\mathbf{O}\mathbf{V}^\top \tag{8}$$

$$d\mathbf{S}_i = d\text{softmax}(d\mathbf{P}_i) = (\text{diag}(\mathbf{P}_i) - \mathbf{P}_i\mathbf{P}_i^\top) d\mathbf{P}_i \tag{9}$$

$$d\mathbf{Q} = d\mathbf{S}\mathbf{K} / \sqrt{d} \tag{10}$$

$$d\mathbf{K} = d\mathbf{S}^\top \mathbf{Q} / \sqrt{d}, \tag{11}$$

As we can see, the backward pass depends on some very large activations from the forward pass. For example, computing  $d\mathbf{V}$  in (7) requires  $\mathbf{P}$ , which are the attention scores of shape `(batch_size, n_heads, seq_len, seq_len)`—the size of this activation matrix depends *quadratically* on the sequence length, explaining the memory issues we encountered above when benchmarking attention at large sequence lengths. During both the forward and backward pass of vanilla attention, we pay significant memory IO costs to transfer  $\mathbf{P}$  and other large activations between on-chip SRAM and GPU HBM. There are *several* such transfers made in standard implementations: for example, a standard backward pass implementation would read  $\mathbf{P}$  from HBM in the computations of both (7) and (9).

The main goal of FlashAttention is to avoid reading and writing the attention matrix to and from HBM, to reduce IO and peak memory costs. We accomplish this using three techniques: tiling, recomputation, and operator fusion.

**Tiling.** To avoid reading and writing the attention matrix to and from HBM, we compute the softmax reduction without access to the whole input. Specifically, we restructure the attention computation to split the input into tiles and make several passes over input tiles, thus incrementally performing the softmax reduction.

**Recomputation.** We avoid storing the large intermediate attention matrices of shape `(batch_size, n_heads, seq_len, seq_len)` in HBM. Instead, we will save certain “activation checkpoints” in HBM and then recompute part of the forward pass during the backward pass, to get the other activations we need for computing gradients. FlashAttention-2 also stores the logsumexp of the attention scores,  $L$ , which will be used to simplify the backward pass computation. The expression for  $L$  is:

$$L_i = \log \left( \sum_j \exp(\mathbf{S}_{ij}) \right) \tag{12}$$

In our final kernel we will compute this in an online manner, but the final result should be the same. With tiling and recomputation together, our memory IO and peak usage no longer depend on `sequence_length`<sup>2</sup> and therefore we may use larger sequence lengths.

**Operator fusion.** Lastly, we avoid repeated memory IO for the attention matrix and other intermediate activations by performing all our operations in a single kernel—this is referred to as operator or kernel fusion. We will write a single Triton kernel for the forward pass that performs all the operations involved in attention with limited data transfer between HBM and SRAM. Operator fusion is partly enabled by recomputation, since we can avoid the usual memory IO we would pay to store every intermediate activation to HBM.

For more intuition on these techniques, check out the FlashAttention papers [Dao et al., 2022, Dao, 2023].

**Backward pass with recomputation.** Using  $L$ , we can do the appropriate recomputation and compute the backward pass efficiently. Before we start the backward pass, we pre-compute into global memory the value  $D = \text{rowsum}(\mathbf{O} \circ \mathbf{dO})$  (where  $\circ$  is element-wise multiplication), which is equal to  $\text{rowsum}(\mathbf{P} \circ \mathbf{dP})$  since  $\mathbf{P}\mathbf{dP}^\top = \mathbf{P}(\mathbf{dO}\mathbf{V}^\top)^\top = (\mathbf{P}\mathbf{V})\mathbf{dO}^\top = \mathbf{O}\mathbf{dO}^\top$  (and  $\text{rowsum}(\mathbf{A} \circ \mathbf{B}) = \text{diag}(\mathbf{A}\mathbf{B}^\top)$  for any matrices  $\mathbf{A}$  and  $\mathbf{B}$ ). With the  $L$  and  $D$  vectors, the backward pass can be computed without softmax. The full calculation for the backward pass is now:

$$\mathbf{S} = \mathbf{Q}\mathbf{K}^\top / \sqrt{d} \tag{13}$$

$$\mathbf{P}_{ij} = \exp(\mathbf{S}_{ij} - L_i) \tag{14}$$

$$\mathbf{dV} = \mathbf{P}^\top \mathbf{dO} \tag{15}$$

$$\mathbf{dP} = \mathbf{dO}\mathbf{V}^\top \tag{16}$$

$$\mathbf{dS}_{ij} = \mathbf{P}_{ij} \circ (\mathbf{dP}_{ij} - D_i) \tag{17}$$

$$\mathbf{dQ} = \mathbf{dS}\mathbf{K} / \sqrt{d} \tag{18}$$

$$\mathbf{dK} = \mathbf{dS}^\top \mathbf{Q} / \sqrt{d}, \tag{19}$$

We can see that the sequence of operations does not require us to have stored the attention scores  $\mathbf{P}$  in HBM during the forward pass—we recompute them from the activations  $\mathbf{Q}$ ,  $\mathbf{K}$ , and  $L$  in (13) and (14).

**Details of the flash attention forward pass.** Now that we have a high level idea of the techniques used in FlashAttention-2, we will dive into the details of the FA2 forward pass kernel that you will implement. In order to avoid reading and writing the attention matrix to and from HBM, we wish to use tiling, i.e., computing each tile of the output independently of the others. This requires us to be able to compute tiles of  $P$ , ideally tiled in both dimensions (for queries and for keys).

However, when we apply softmax to  $S$ , we require entire rows of  $S$  to be reduced to compute the softmax denominator, meaning we cannot compute  $P$  in tiles directly. FlashAttention-2 solves this problem using *online softmax*. In the following text, we will use subscript index  $i$  to denote the current query tile, and superscript index  $(j)$  to denote the current key tile. The tiles along the query dimension will be of size  $B_q$ , and the key dimension,  $B_k$ . We will not tile along the hidden dimension  $d$ .

We also keep some row-wise running values,  $m_i^{(j)} \in \mathbb{R}^{B_q}$  and  $l_i^{(j)} \in \mathbb{R}^{B_q}$ . The row-wise  $m_i^{(j)}$  value is a running maximum, which is tracked so we can compute softmax in a numerically stable manner (recall this trick from our softmax implementation in Assignment 1). We will update  $m_i^{(j)}$  with each new row-wise tile of  $S$  (when  $j$  increases). Using the running maximum, we can compute the unnormalized softmax values (numerators) as  $\tilde{\mathbf{P}}_i^{(j)} = \exp(\mathbf{S}_{ij} - m_i^{(j)})$ .  $l_i^{(j)}$  is a running proxy for the softmax denominator, and will be updated using the unnormalized softmax values as  $l_i^{(j)} = \exp(m_i^{(j-1)})$ . When we finally write the output, we will need to finish normalizing it by using  $l_i^{(T_k)}$ , which is the final value of  $l_i^{(j)}$  after processing all key tiles. Algorithm 1 shows the forward pass as it should be implemented on GPU.

---

**Algorithm 1** FlashAttention-2 forward pass

---

**Require:**  $\mathbf{Q} \in \mathbb{R}^{N_q \times d}$ ,  $\mathbf{K}, \mathbf{V} \in \mathbb{R}^{N_k \times d}$ , tile sizes  $B_q, B_k$

Split  $\mathbf{Q}$  into  $T_q = \left\lceil \frac{N_q}{B_q} \right\rceil$  tiles  $\mathbf{Q}_1, \dots, \mathbf{Q}_{T_q}$  of size  $B_q \times d$

Split  $\mathbf{K}, \mathbf{V}$  into  $T_k = \left\lceil \frac{N_k}{B_k} \right\rceil$  tiles  $\mathbf{K}^{(1)}, \dots, \mathbf{K}^{(T_k)}$  and  $\mathbf{V}^{(1)}, \dots, \mathbf{V}^{(T_k)}$  of size  $B_k \times d$

**for**  $i = 1, \dots, T_q$  **do**

  Load  $\mathbf{Q}_i$  from global memory

  Initialize  $\mathbf{O}_i^{(0)} = \mathbf{0} \in \mathbb{R}^{B_q \times d}$ ,  $l_i^{(0)} = 0 \in \mathbb{R}^{B_q}$ ,  $m_i^{(0)} = -\infty \in \mathbb{R}^{B_q}$

**for**  $j = 1, \dots, T_k$  **do**

    Load  $\mathbf{K}^{(j)}, \mathbf{V}^{(j)}$  from global memory

    Compute tile of pre-softmax attention scores  $\mathbf{S}_i^{(j)} = \frac{\mathbf{Q}_i (\mathbf{K}^{(j)})^\top}{\sqrt{d}} \in \mathbb{R}^{B_q \times B_k}$

    Compute  $m_i^{(j)} = \max \left( m_i^{(j-1)}, \text{rowmax} \left( \mathbf{S}_i^{(j)} \right) \right) \in \mathbb{R}^{B_q}$

    Compute  $\tilde{\mathbf{P}}_i^{(j)} = \exp \left( \mathbf{S}_i^{(j)} - m_i^{(j)} \right) \in \mathbb{R}^{B_q \times B_k}$

    Compute  $l_i^{(j)} = \exp \left( m_i^{(j-1)} - m_i^{(j)} \right) l_i^{(j-1)} + \text{rowsum} \left( \tilde{\mathbf{P}}_i^{(j)} \right) \in \mathbb{R}^{B_q}$

    Compute  $\mathbf{O}_i^{(j)} = \text{diag} \left( \exp \left( m_i^{(j-1)} - m_i^{(j)} \right) \right) \mathbf{O}_i^{(j-1)} + \tilde{\mathbf{P}}_i^{(j)} \mathbf{V}^{(j)}$

**end for**

  Compute  $\mathbf{O}_i = \text{diag} \left( l_i^{(T_k)} \right)^{-1} \mathbf{O}_i^{(T_k)}$

  Compute  $L_i = m_i^{(T_k)} + \log \left( l_i^{(T_k)} \right)$

  Write  $\mathbf{O}_i$  to global memory as the  $i$ -th tile of  $\mathbf{O}$ .

  Write  $L_i$  to global memory as the  $i$ -th tile of  $L$ .

**end for**

Return the output  $\mathbf{O}$  and the logsumexp  $L$ .

---

Before we get into implementing the forward pass in Triton, we collect here a few general tips and tricks for writing Triton kernels.

### Triton Tips and Tricks

---

- You can use print statements in Triton with `tl.device_print` to debug: [https://triton-lang.org/main/python-api/generated/triton.language.device\\_print.html](https://triton-lang.org/main/python-api/generated/triton.language.device_print.html). There is a setting `TRITON_INTERPRET=1` to run the Triton interpreter on CPU, though we have found it buggy.
- When defining block pointers, make sure they have the correct offsets, and that block offsets are multiplied by the appropriate tile sizes.
- The launch grid of thread blocks is set with

```
kernel_fn[(launch_grid_d1, launch_grid_d2, ...)](...arguments...)
```

in the methods of the `torch.autograd.Function` subclass, as we saw in the weighted sum example.

- Perform matrix multiplications with `tl.dot`.
- To advance a block pointer, use `*_block_ptr = *_block_ptr.advance(...)`

### Problem (flash\_forward): 25 points

---

- (a) Write a pure PyTorch (no Triton) `autograd.Function` that implements the FlashAttention-2 forward pass. This will be a lot slower than the regular PyTorch implementation, but will help you debug your Triton kernel.

Your implementation should take input  $\mathbf{Q}$ ,  $\mathbf{K}$ , and  $\mathbf{V}$  as well as a flag `is_causal` and produce the output  $\mathbf{O}$  and the logsumexp value  $L$ . You can ignore the `is_causal` flag for this task. The `autograd.Function` forward should then use `save L, Q, K, V, O` for the backward pass and return `O`. Remember that the implementation of the `forward` method of `autograd.Function` always takes the context as its first parameter. Any `autograd.Function` class needs to implement a `backward` method, but for now you can make it just raise `NotImplementedError`. If you need something to compare against, you can implement Equation 4 to 6 and 12 in PyTorch and compare your outputs.

The interface is then `def forward(ctx, Q, K, V, is_causal=False)`. Determine your own tile sizes, but make sure they are at least of size  $16 \times 16$ . We will always test your code with dimensions that are clean powers of 2 and at least 16, so you don't need to worry about out-of-bounds accesses.

**Note that your implementation must be batched.** The batch dimension should be a leading dimension in your computation. For this part, all of your operations should be executed along the batch dimension too. That is, if doing an operation over a tile, you might access a tensor like `[:, tile_start:tile_end, :]`

**Deliverable:** A `torch.autograd.Function` subclass that implements FlashAttention-2 in the forward pass. To test your code, implement `[adapters.get_flashattention_autograd_function_pytorch]`. Then, run the test with `uv run pytest -k test_flash_forward_pass_pytorch` and make sure your implementation passes it.

(b) Write a Triton kernel for the forward pass of FlashAttention-2 following Algorithm 1. Then, write another subclass of `torch.autograd.Function` that calls this (fused) kernel in the forward pass, instead of computing the result in PyTorch. A few problem-specific tips:

- To debug, we suggest comparing the results of each Triton operation you perform with the tiled PyTorch implementation you wrote in part (a).
- Your launch grid should be set as  $(T_q, \text{batch\_size})$ , meaning each Triton program instance will load only elements from a single batch index, and only read/write to a single query tile of  $\mathbf{Q}$ ,  $\mathbf{O}$ , and  $L$ .
- The kernel should only have a single loop, which will iterate key tiles  $1 \leq j \leq T_k$ .
- Advance block pointers at the end of the loop.
- Use the function declaration below (using the block pointer we give you, you should be able to infer the setup of the rest of the pointers):

```

1  @triton.jit
2  def flash_fwd_kernel(
3      Q_ptr, K_ptr, V_ptr,
4      O_ptr, L_ptr,
5      stride_qb, stride_qq, stride_qd,
6      stride_kb, stride_kk, stride_kd,
7      stride_vb, stride_vk, stride_vd,
8      stride_ob, stride_oq, stride_od,
9      stride_lb, stride_lq,
10     N_QUERIES, N_KEYS,
11     scale,
12     D: tl.constexpr,
13     Q_TILE_SIZE: tl.constexpr,
14     K_TILE_SIZE: tl.constexpr,
15 ):
16     # Program indices
17     query_tile_index = tl.program_id(0)
18     batch_index = tl.program_id(1)
19
20     # Offset each pointer with the corresponding batch index
21     # multiplied with the batch stride for each tensor
22     Q_block_ptr = tl.make_block_ptr(
23         Q_ptr + batch_index * stride_qb,
24         shape=(N_QUERIES, D),
25         strides=(stride_qq, stride_qd),
26         offsets=(query_tile_index * Q_TILE_SIZE, 0),
27         block_shape=(Q_TILE_SIZE, D),
28         order=(1, 0),
29     )
30
31     ...

```

where `scale` is  $\frac{1}{\sqrt{d}}$  and `Q_TILE_SIZE` and `K_TILE_SIZE` are  $B_q$  and  $B_k$  respectively. You can tune these later.

These additional guidelines may help you avoid precision issues:

- The on chip buffers  $(\mathbf{O}_i, l, m)$  should have `dtype tl.float32`. If you're accumulating into an output buffer, use the `acc` argument (`acc = tl.dot(..., acc=acc)`).
- Cast  $\tilde{\mathbf{P}}_i^{(j)}$  to the `dtype` of  $\mathbf{V}^{(j)}$  before multiplying them, and cast  $\mathbf{O}_i$  to the appropriate `dtype` before writing it to global memory. Casting is done with `tensor.to`. You can get the `dtype` of a tensor with `tensor.dtype`, and the `dtype` of a block pointer/pointer with `*_block_ptr.type.element_ty`.

**Deliverable:** A `torch.autograd.Function` subclass that implements FlashAttention-2 in the forward pass using your Triton kernel. Implement `[adapters.get_flash_autograd_function_triton]`. Then, run the test with `uv run pytest -k test_flash_forward_pass_triton` and make sure your implementation passes it. **Note that this is a parameterized test that runs with the `is_causal` flag set to `False` and set to `True`; you should only pass the `False` version at this stage.**

- (c) Add a flag as the last argument to your `autograd.Function` implementation for causal masking. This should be a boolean flag that when set to `True` enables an index comparison for causal masking. Your Triton kernel should have a corresponding additional parameter `is_causal: tl.constexpr` (this is a required type annotation). In Triton, construct appropriate index vectors for queries and keys, and compare them to form a square mask of size  $B_q \times B_k$ . For elements that are masked out, add the constant value of `-1e6` to the corresponding elements of the attention score matrix  $S_i^{(j)}$ . Make sure to save the mask flag for backward using `ctx.is_causal = is_causal`.

**Deliverable:** An additional flag for your `torch.autograd.Function` subclass that implements the FlashAttention-2 forward pass with causal masking using your Triton kernel. Make sure that the flag is optional with default `False` so the previous tests still pass. You should now pass both versions of `test_flash_forward_pass_triton`.

**Implementing the backward pass with recomputation** Notice that unlike the standard backward pass in Eq 7 to 11, we can use recomputation to avoid the softmax operation in the backward pass shown in Eq 13 to 19. This means that we can compute the backward pass using a trivial kernel, and no online tricks are required. Thus, for this part, you can implement backward by calling `torch.compile` on a regular PyTorch function (not Triton).

#### Problem (flash\_backward): 10 points

---

Implement the backward pass for your FlashAttention-2 `autograd.Function` using PyTorch (not Triton) and `torch.compile`. Your implementation should take the `Q`, `K`, `V`, `O`, `dO`, and `L` tensors as output, and return `dQ`, `dK` and `dV`. Remember to compute and use the `D` vector. You may follow along the computations of Equations 13 to 19.

You will want this code somewhere that it can be imported into both your basic PyTorch implementation as well as your Triton implementation.

**Deliverable:** To test your implementation, run `uv run pytest -k test_flash_backward`.

Let's now compare the performance of your (partially) Triton implementation of FlashAttention-2 with your PyTorch implementation of regular Attention.

#### Problem (flash\_benchmarking): 15 points

---

- (a) Write a benchmarking script using `triton.testing.do_bench` that compares the performance of your (partially) Triton implementation of FlashAttention-2 forward and backward passes with a regular PyTorch implementation (i.e., not using FlashAttention).

Specifically, you will report a table that includes latencies for forward, backward, and the end-to-end forward-backward pass, for both your Triton and PyTorch implementations. Randomly generate any necessary inputs before you start benchmarking, and run the benchmark on a single GPU; report the kind of GPU used in your writeup. Always use batch size 1 and causal masking. Sweep over the cartesian product of sequence lengths of various powers of 2 from 128 up to

65536, embedding dimension sizes of various powers of 2 from 16 up to size 128, and precisions of `torch.bfloat16` and `torch.float32`. You will likely need to adjust tile sizes depending on the input sizes.

**Deliverable:** A table of results comparing your implementation of FlashAttention-2 with the PyTorch implementation, using the settings above and reporting forward, backward, and end-to-end latencies.

### 1.3.3 [Optional] Improving FlashAttention-2 Further

We challenge you to further improve the performance of your implementation, using any tricks you can come up with. The restrictions are that you cannot change the input/outputs of the function, and you must use Triton (no CUDA). Your inputs should use BF16 with causal masking, and it must pass the same tests as your regular implementation. The implementation must also be your own, and you cannot use pre-existing implementations. The test to verify your implementation is the following:

```
1 def test_timing_flash_forward_backward():
2     n_heads = 16
3     d_head = 64
4     sequence_length = 16384
5     q, k, v = torch.randn(
6         3, n_heads, sequence_length, d_head, device='cuda', dtype=torch.bfloat16, requires_grad=True
7     )
8
9     flash = torch.compile(FlashAttention2.apply)
10
11     def flash_forward_backward():
12         o = flash(q, k, v, True)
13         loss = o.sum()
14         loss.backward()
15
16     results = triton.testing.do_bench(flash_forward_backward, rep=10000, warmup=1000)
17     print(results)
```

For testing purposes, you can reduce the repetition and warmup time (given in ms) to a something shorter. Some ideas for improvement:

- Tune the tile sizes for your kernel (use Triton autotune for this!)
- Tune additional Triton config parameters
- Implement the backward pass in Triton, not just `torch.compile` (see Section 1.3.4 below)
- Do two passes over your input for the backward pass, one for  $dQ$  and another for  $dK$  and  $dV$  to avoid atomics or synchronization between blocks.
- Stop program instances early when doing causal masking, skipping all tiles that are always all zero
- Separate the non-masked tiles from the tile diagonals, computing the first without ever comparing indices, and the second with a single comparison
- Use TMA (Tensor Memory Accelerator) functionality on H100, following a similar pattern to this tutorial.

### 1.3.4 OPTIONAL: Triton backward pass

If you're interested in getting more practice with Triton, we provide the tiled FlashAttention-2 backward pass below which you can implement in Triton. Algorithm 2 shows the FlashAttention-2 backward pass as it should be implemented in Triton. A key trick here is to compute  $\mathbf{P}$  twice, once for the backward pass for  $\mathbf{dQ}$  and another time for  $\mathbf{dK}$  and  $\mathbf{dV}$ . This lets us skip synchronization across thread blocks.

---

#### Algorithm 2 Tiled FlashAttention-2 backward pass

---

**Require:**  $\mathbf{Q}, \mathbf{O}, \mathbf{dO} \in \mathbb{R}^{N_q \times d}$ ,  $\mathbf{K}, \mathbf{V} \in \mathbb{R}^{N_k \times d}$ ,  $L \in \mathbb{R}^{N_q}$ , tile sizes  $B_q, B_k$

Compute  $D = \text{rowsum}(\mathbf{dO} \circ \mathbf{O}) \in \mathbb{R}^{N_q}$

Split  $\mathbf{Q}, \mathbf{O}, \mathbf{dO}$  into  $T_q = \left\lceil \frac{N_q}{B_q} \right\rceil$  tiles  $\mathbf{Q}_1, \dots, \mathbf{Q}_{T_q}$ ,  $\mathbf{O}_1, \dots, \mathbf{O}_{T_q}$ ,  $\mathbf{dO}_1, \dots, \mathbf{dO}_{T_q}$ , each of size  $B_q \times d$

Split  $\mathbf{K}, \mathbf{V}$  into  $T_k = \left\lceil \frac{N_k}{B_k} \right\rceil$  tiles  $\mathbf{K}^{(1)}, \dots, \mathbf{K}^{(T_k)}$  and  $\mathbf{V}^{(1)}, \dots, \mathbf{V}^{(T_k)}$ , each of size  $B_k \times d$

Split  $L, D$  into  $T_q$  tiles  $L_1, \dots, L_{T_q}$  and  $D_1, \dots, D_{T_q}$ , each of size  $B_q$

**for**  $j = 1, \dots, T_k$  **do**

  Load  $\mathbf{K}^{(j)}, \mathbf{V}^{(j)}$  from global memory

  Initialize  $\mathbf{dK}^{(j)} = \mathbf{dV}^{(j)} = \mathbf{0} \in \mathbb{R}^{B_k \times d}$

**for**  $i = 1, \dots, T_q$  **do**

    Load  $\mathbf{Q}_i, \mathbf{O}_i, \mathbf{dO}_i, \mathbf{dQ}_i$  from global memory

    Compute tile of attention scores  $\mathbf{S}_i^{(j)} = \frac{\mathbf{Q}_i (\mathbf{K}^{(j)})^\top}{\sqrt{d}} \in \mathbb{R}^{B_q \times B_k}$

    Compute attention probabilities  $\mathbf{P}_i^{(j)} = \exp(\mathbf{S}_i^{(j)} - L_i) \in \mathbb{R}^{B_q \times B_k}$

    Compute  $\mathbf{dV}^{(j)} += (\mathbf{P}_i^{(j)})^\top \mathbf{dO}_i \in \mathbb{R}^{B_k \times d}$

    Compute  $\mathbf{dP}_i^{(j)} = \mathbf{dO}_i \mathbf{V}_j^\top \in \mathbb{R}^{B_q \times B_k}$

    Compute  $\mathbf{dS}_i^{(j)} = \mathbf{P}_i^{(j)} \circ (\mathbf{dP}_i^{(j)} - D_i) / \sqrt{d} \in \mathbb{R}^{B_q \times B_k}$

    Load  $\mathbf{dQ}_i$  from global memory, then update  $\mathbf{dQ}_i += \mathbf{dS}_i^{(j)} \mathbf{K}^{(j)} \in \mathbb{R}^{B_q \times d}$ , and write back to global memory. Must be atomic for correctness!

    Compute  $\mathbf{dK}^{(j)} += (\mathbf{dS}_i^{(j)})^\top \mathbf{Q}_i \in \mathbb{R}^{B_k \times d}$ .

**end for**

  Write  $\mathbf{dK}^{(j)}$  and  $\mathbf{dV}^{(j)}$  to global memory as the  $j$ -th tiles of  $\mathbf{dK}$  and  $\mathbf{dV}$ .

**end for**

Return  $\mathbf{dQ}, \mathbf{dK}, \mathbf{dV}$ .

---

## References

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